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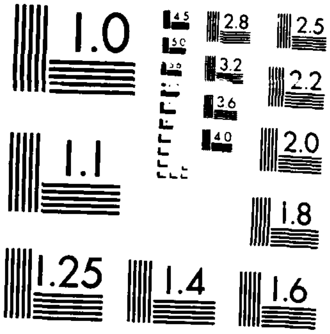

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The Propagation of Electromagnetic Waves in Thin Dielectric Slabs

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This report presents the solutions of Maxwell's equations for the TE and TM modes of propagation along a thin dielectric slab. From these, the propagation constants are determined and the electric and magnetic field patterns are plotted. These calculations are done with the use of the dielectric coefficients of $LiNO_3$ and GaAs. The wavelengths used are 0.82 μm (diode laser), 1.064 μm (Nd:YAG laser), and 1.55 μm (diode laser). The thicknesses of the slabs range from 0.1 to 1.0 μm.

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1. Introduction

Because dielectric waveguides are frequently used as part of the experimental setups in the laboratory, it is of interest to have general programs that calculate their characteristics. These waveguides generally consist of thin sheets (a few wavelengths or less) of a low-loss nonmagnetic dielectric material. The electromagnetic wave is launched at one end of the slab and guided to the sample [1].

In this report, we idealize the situation to a single slab of infinite length in two dimensions, and a thickness, 2d, which will be arbitrary. First, we introduce the appropriate Maxwell's equations for the problem and derive the wave equation for the transverse electric (TE) and transverse magnetic (TM) modes. We obtain the general solutions for both modes inside and outside the dielectric and, by satisfying the boundary conditions at the surfaces of the slab, obtain four transcendental equations which provide the solutions to the problem. There is one equation for each mode, the TE even, TE odd, TM even, and TM odd. Next, we assign values to our parameters and numerically solve the transcendental equations. This procedure is repeated for several thicknesses and indices of refraction. The results are presented in graphical and tabular form. The appendix contains the listings of the programs used.

2. Maxwell's Equations

Our waveguide extends infinitely in the y and z directions, and from $-d$ to d in the x direction (see fig. 1a). The wave is launched in the y direction. The relevant Maxwell equations for the problem (in centimeter-gram-seconds) are

$$\nabla \times \vec{E} = ik\vec{H} \tag{1}$$

and

$$-\nabla \times \vec{H} = ik\varepsilon\vec{E} \ , \tag{2}$$

where ε is the dielectric constant. The time dependence is assumed to be exp(−iωt), with $k = \omega/c$. Throughout the problem, μ is considered constant and equal to one. In all cases considered, we assume that none of the fields vary in the z direction ($\partial/\partial z = 0$).

In our first case, the TE mode, $E_x = 0$, $E_y = 0$, and $E_z \neq 0$. So, Maxwell's equations yield

$$ikH_x = \frac{\partial}{\partial y} E_z \ , \tag{3}$$

$$ikH_y = -\frac{\partial}{\partial x} E_z \ , \tag{4}$$

and

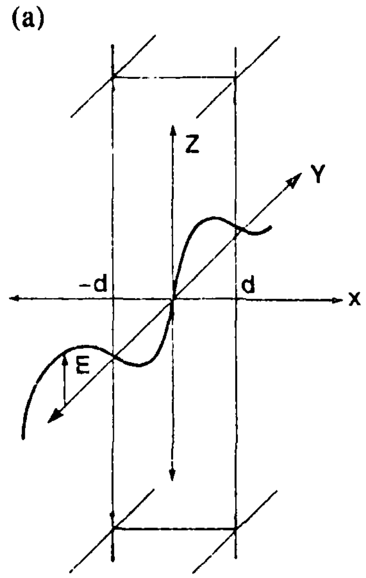
$$\frac{\partial}{\partial y} H_x - \frac{\partial}{\partial x} H_y = ik\varepsilon E_z \ . \tag{5}$$

Combining these equations gives us the wave equation

$$\nabla_t^2 E_z = -k^2 \varepsilon E_z \qquad \begin{array}{ll} \varepsilon = \varepsilon & |x| < d \\ \varepsilon = 1 & |x| > d \ , \end{array} \tag{6}$$

where $\nabla_t^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$.

Figure 1. Physical representation of problem with (a) transverse electric field and (b) transverse magnetic field.



We will assume a dependence in the y direction of the form $\exp(i\beta y)$. We will also assume odd and even solutions of the form $\sin k_1 x$ and $\cos k_1 x$ inside the dielectric, and a decaying exponential, $\exp(-\Gamma x)$, outside the dielectric. For guided waves in a lossless material, we must have Γ real and positive.

For even solutions, using equation (6), we have

$$E_z = E_1 \cos k_1 x e^{i\beta y} \qquad |x| < d \ , \tag{7}$$

$$E_z = E_2 e^{-\Gamma x} e^{i\beta y} \qquad |x| > d \ , \tag{8}$$

where

$$k_1^2 + \beta^2 = k^2 \varepsilon \ , \tag{9}$$

$$-\Gamma^2 + \beta^2 = k^2 \ ; \tag{10}$$

therefore,

$$\Gamma = \sqrt{k^2(\varepsilon - 1) - k_1^2} \ . \tag{11}$$

We know that E_z and H_y must be continuous at the boundaries $x = d$ and $x = -d$. We need only solve at one of these boundaries, though, because

of the symmetry of the problem. We can find H_y by using equation (4) with equations (7) and (8):

$$H_y = \frac{1}{ik} E_1 k_1 \sin k_1 x e^{i\beta y} \qquad |x| < d \ , \tag{12}$$

$$H_y = \frac{1}{ik} \Gamma E_2 e^{-\Gamma x} e^{i\beta y} \qquad |x| > d \ . \tag{13}$$

Setting equation (7) equal to (8), and (12) equal to (13) gives us the system of equations

$$E_1 \cos k_1 d = E_2 e^{-\Gamma d} \ , \tag{14}$$

$$E_1 k_1 \sin k_1 d = E_2 \Gamma e^{-\Gamma d} \ , \tag{15}$$

which is reduced to

$$k_1 \tan k_1 d = \Gamma \ . \tag{16}$$

Throughout the remainder of our discussion, we will be using the following substitutions:

$$u = k_1 d \tag{17}$$

and

$$A^2 = k^2 (\varepsilon - 1) d^2 \ . \tag{18}$$

Using equations (11), (17), and (18), equation (16) becomes

$$\tan u = \frac{\sqrt{A^2 - u^2}}{u} \ . \tag{19}$$

This transcendental equation will be solved in section 3.

For odd solutions, sine replaces cosine in the material, making our wave equation reduce to

$$E_z = E_3 \sin k_1 x e^{i\beta y} \qquad |x| < d \ , \tag{20}$$

$$E_z = E_4 e^{-\Gamma x} e^{i\beta y} \qquad |x| > d \ . \tag{21}$$

Equations (9), (10), and (11) still hold. Using equation (4) on equations (20) and (21) yields

$$H_y = -\frac{1}{ik} E_3 k_1 \cos k_1 x\, e^{i\beta y} \qquad |x| < d \ , \tag{22}$$

$$H_y = \frac{1}{ik} E_4 \Gamma\, e^{-\Gamma x} e^{i\beta y} \qquad |x| > d \ . \tag{23}$$

At $x = d$, we have a system of equations which produces

$$-\cot k_1 d = \frac{\Gamma d}{k_1 d} \ . \tag{24}$$

Using our substitutions, from equations (11), (17), and (18) we get

$$-\cot u = \frac{\sqrt{A^2 - u^2}}{u} \ . \tag{25}$$

This transcendental equation will also be solved in section 3.

The calculations for the TM modes are very similar. We use the same Maxwell's equations, but now the electromagnetic wave is described by figure 1b:

$$H_x = 0, H_y = 0, H_z \neq 0 \ .$$

Maxwell's equations yield

$$E_x = -\frac{1}{ik\varepsilon} \frac{\partial}{\partial y} H_z \ , \tag{26}$$

$$E_y = \frac{1}{ik\varepsilon} \frac{\partial}{\partial x} H_z \ , \tag{27}$$

$$H_z = \frac{1}{ik} \left(\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \right) \ . \tag{28}$$

Combining these equation- yields the wave equation

$$\nabla_t^2 H_z = -k^2 \varepsilon H_z \ . \tag{29}$$

For even solutions, we assume

$$H_z = H_1 \cos k_1 x\, e^{i\beta y} \qquad |x| < d \ , \tag{30}$$

$$H_z = H_2\, e^{-\Gamma x} e^{i\beta y} \qquad |x| > d \ , \tag{31}$$

where k_1 and Γ are given in equations (9) and (10). E_y and H_z must be continuous over x. Using equation (27) with equations (30) and (31) gives us a system of equations at $x = d$ which yields

$$\tan k_1 d = \frac{\varepsilon\Gamma}{k_1} \ . \tag{32}$$

Or, after making our usual substitutions, with equations (11), (17), and (18), we get

$$\tan u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u} \ . \tag{33}$$

This will be numerically solved in section 3 with the other transcendental equations.

For the TM odd mode, a very similar set of calculations arrives at the solution

$$-\cot u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u} \ . \tag{34}$$

We now have the four transcendental equations providing the even and odd solutions to the TE and TM modes.

3. Numerical Calculation of Solutions

In section 2 we obtained analytical solutions to our problem in the form of these four transcendental equations:

$$\text{TE even} \qquad \tan u = \frac{\sqrt{A^2 - u^2}}{u}$$

$$\text{TE odd} \qquad -\cot u = \frac{\sqrt{A^2 - u^2}}{u}$$

$$\text{TM even} \qquad \tan u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u}$$

$$\text{TM odd} \qquad -\cot u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u} \quad .$$

We wish to numerically calculate all the solutions to these equations using the Newton-Raphson method. We must first assign values to A and ε. To assign a value to A we must first break it down to its components. We assume the following:

$$n = 3.5 \qquad \text{(GaAs)}$$

$$\varepsilon = 12.25 = n^2 \qquad \text{(GaAs)}$$

$$d = 0.175\ \mu\text{m}$$

$$\lambda = 1.064\ \mu\text{m}$$

$$k = 5.93 \times 10^4\ \mu\text{m}$$

$$A = 3.466 = \sqrt{k^2(\varepsilon - 1)d^2} \quad .$$

These values are chosen to illustrate the method of solution to be described, but are not to represent practical values in use in the laboratory [2]. Although there may be some use of waveguides as thin as this, most optical waveguides are in the neighborhood of 1 µm.

Just knowing these parameters is not enough. We must know the number of solutions, and their approximate values, for the Newton-Raphson method to work well. Figure 2a is a plot of tan u, $-$cot u, $(A^2 - u^2)^{1/2}/u$, and $\varepsilon(A^2 - u^2)^{1/2}/u$ versus u. The intersections of the trigonometric func-

tions with the latter two functions are the solutions to our problem. Their exact values, so far, are unknown, but they provide a good first guess for the program, as well as a check on the number and range of our solutions. Our initial estimate from figure 2a for the first even solutions was $\pi/2 - d$. For each successive even solution, our initial guess was the previous final approximation plus π. For the first odd solutions, our initial guess was $\pi - d$. We used the same method for successive solutions (of which there were none) as in the even case. The markers on figure 2a are the values of the solutions generated by the program with the Newton-Raphson algorithm. Their "u" value is given in table 1.

We now wish to see the result of altering our input parameters--thickness, dielectric coefficient, and wavelength. Table 2 shows values of u for all possible solutions given these parameters. All combinations were used for thicknesses of 0.1, 0.35, and 1.00 µm, and wavelengths of 0.820, 1.064, and 1.550 µm, with a dielectric coefficient of 12.25. We also did one trial with a dielectric coefficient of 4.80 ($LiNO_3$), a thickness of 1.00 µm, and a wavelength of 1.064 µm.

We can see from this table that increasing the wavelength, decreasing the thickness, or decreasing the dielectric coefficient will decrease the number of solutions. However, the first even modes (TE and TM) will never disappear. If the value of A drops below $\pi/2$, there will be no odd solutions, and only one of each even solution. If the value of A is very high, the number of solutions will be approximately $4 \times A/\pi$ (A/π of each type). The separation of the u's for any two consecutive solutions of the same type approaches π for the lower value modes. These results are illustrated in figures 2a through j. More detailed effects of altering the input parameters are shown in the next section.

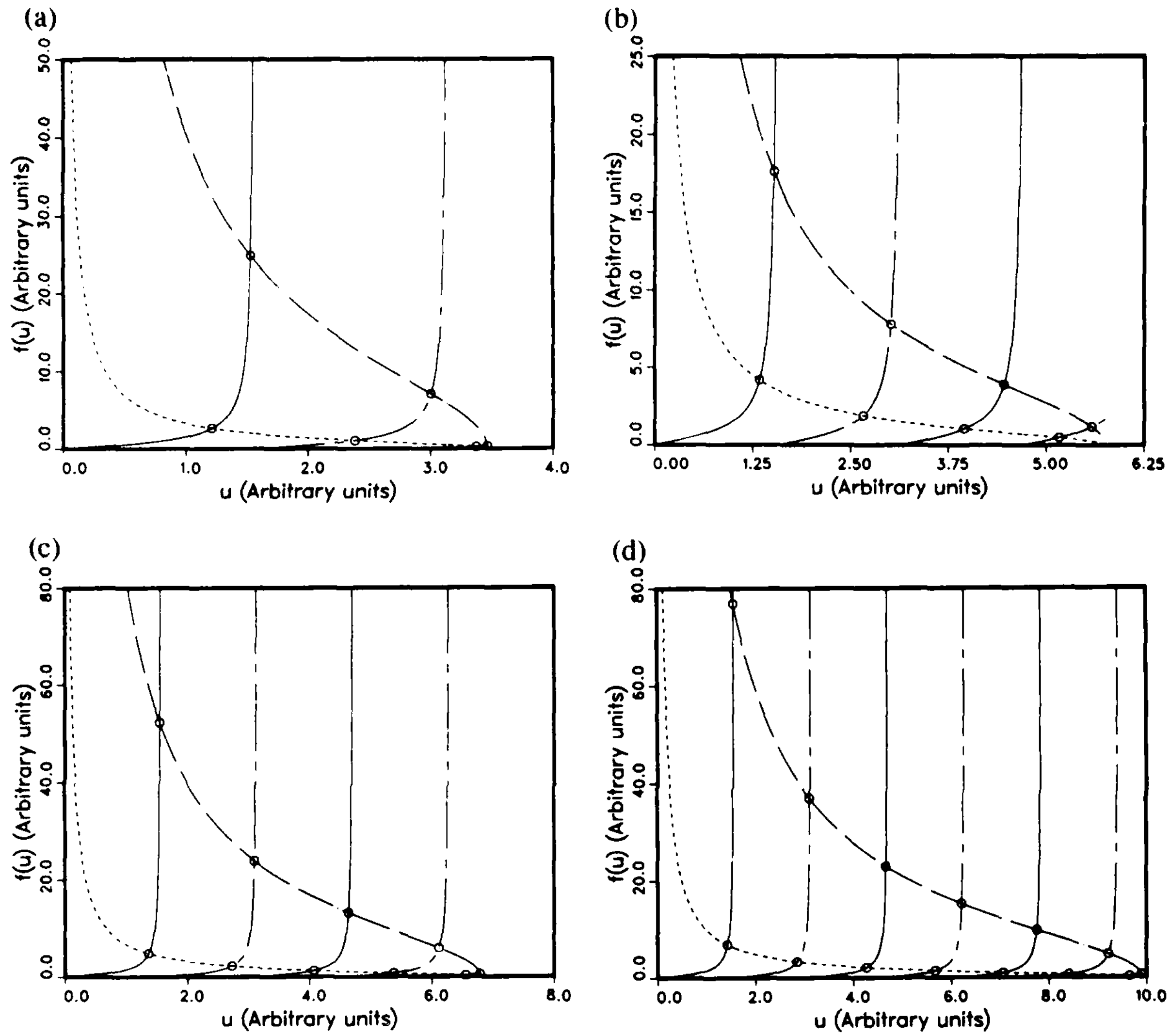
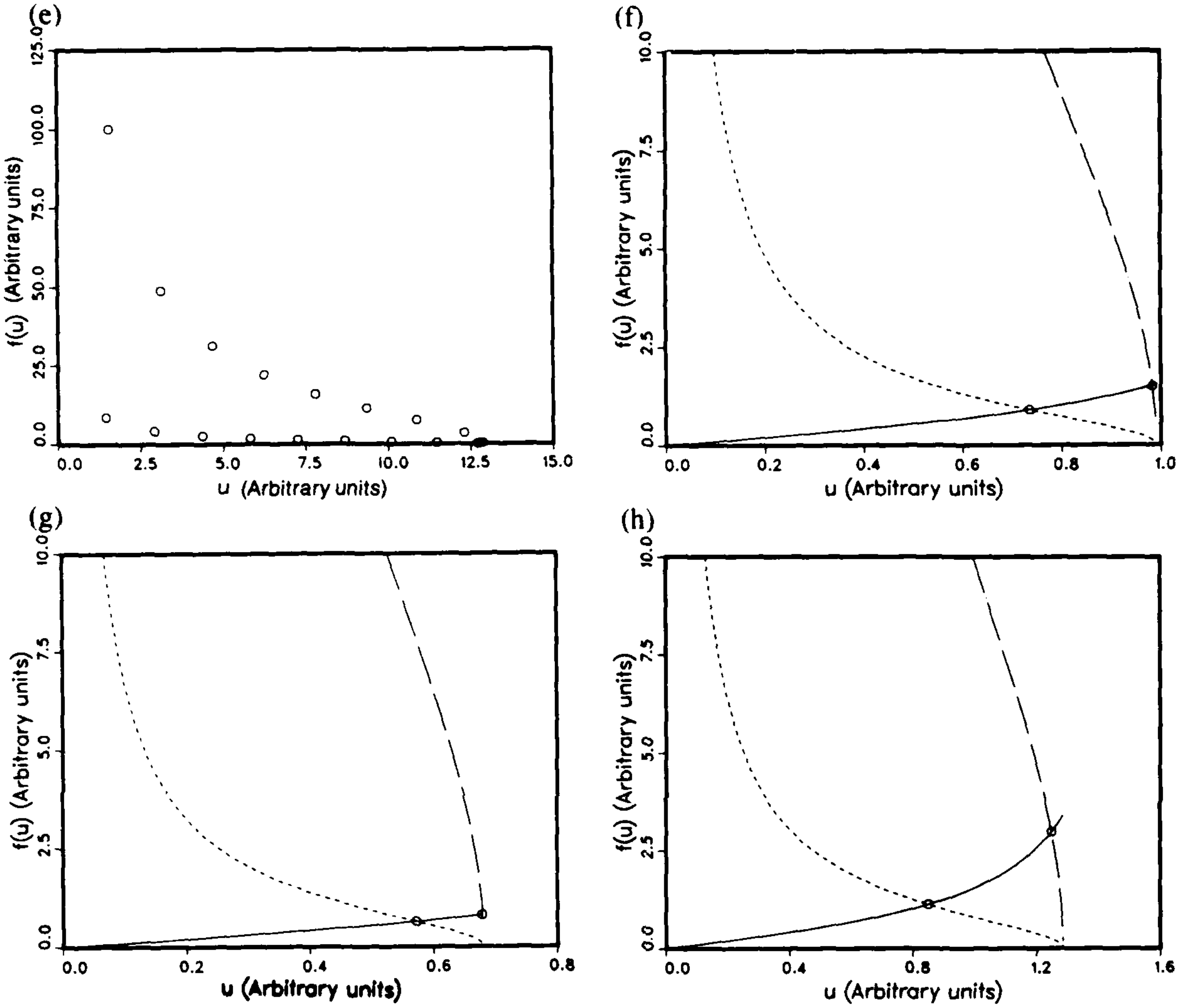


Figure 2. Plots of both sides of four transcendental equations (19, 25, 33, 34) presented in text. Circles mark points derived by Newton-Raphson approximation. These points define parameters for viable modes of propagation. Calculations were done for a slab with various thicknesses, dielectric coefficients, and free space wavelengths: (a) 0.35-µm slab thickness, 12.25 dielectric coefficient, and 1.064-µm free space wavelength; (b) 1.00-µm slab thickness, 4.80 dielectric coefficient, and 1.064-µm free space wavelength; (c) 1.00-µm slab thickness, 12.25 dielectric coefficient, and 1.550-µm free space wavelength; (d) 1.0-µm slab thickness, 12.25 dielectric coefficient, and 1.064-µm free space wavelength.



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(i) (j)

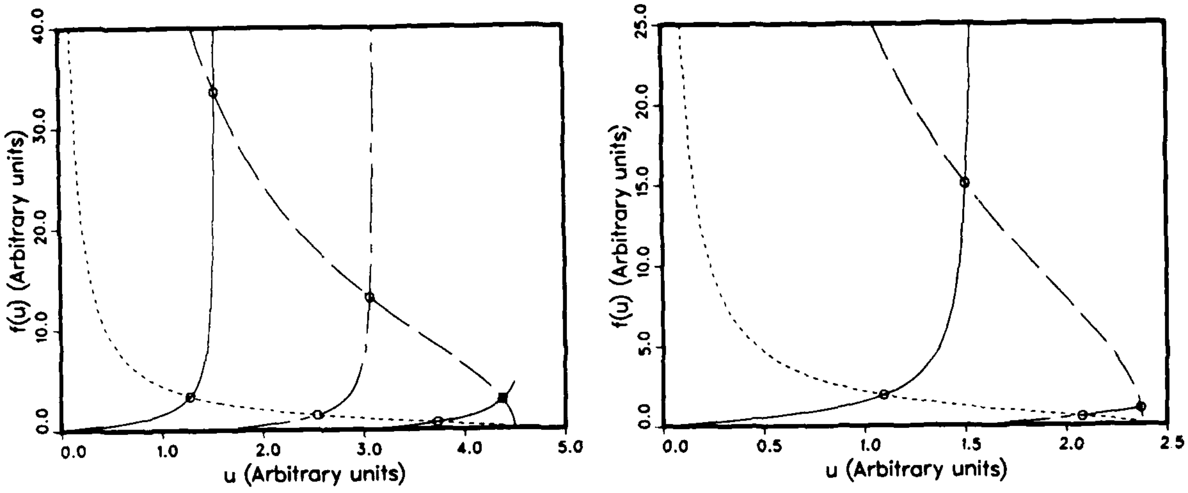

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Table 1. Values of *u* for all possible modes with slab thickness of 0.35 µm (*d* = 0.175 µm), dielectric constant of 12.25, and free space wavelength of 1.064 µm

TEE	TEO	TME	TMO
1.213211536	2.383396864	1.530640006	3.001243353
3.373444796	—	3.464896202	—

Table 2. Values of *u* for all possible modes with physical parameters as specified

Parameters	TEE	TEO	TME	TMO
d = 0.500 μm	1.336460710	2.660983324	1.514054298	3.014218092
ε = 4.80	3.954871893	5.168169022	4.462066174	5.585698605
λ = 1.064 μm	—	—	—	—
d = 0.005 μm	0.849031210	—	1.248315096	—
ε = 12.25	—	—	—	—
λ = 0.820 μm	—	—	—	—
d = 0.050 μm	0.734798610	—	0.982992113	—
ε = 12.25	—	—	—	—
λ = 1.064 μm	—	—	—	—
d = 0.050 μm	0.571712613	—	0.678357184	—
ε = 12.25	—	—	—	—
λ = 1.550 μm	—	—	—	—
d = 0.175 μm	1.281794548	2.541155577	1.541034222	3.065691948
ε = 12.25	3.733202219	—	4.378210068	—
λ = 0.820 μm	—	—	—	—
d = 0.175 μm	1.213211536	2.383396864	1.530640006	3.001243353
ε = 12.25	3.373444796	—	3.464896202	—
λ = 1.064 μm	—	—	—	—
d = 0.175 μm	1.093350649	2.078845978	1.504289746	2.371007681
ε = 12.25	—	—	—	—
λ = 1.550 μm	—	—	—	—
d = 0.500 μm	1.457156897	2.912920237	1.560807824	3.121153116
ε = 12.25	4.365748882	5.813709259	4.680473804	6.237891197
λ = 0.820 μm	7.254138947	8.682868004	7.791815758	9.338629723
	10.092273712	11.464299202	10.866948128	12.302215576
	12.712786674	—	12.846722603	—
d = 0.500 μm	1.426275611	2.849714994	1.557798982	3.114547968
ε = 12.25	4.266947269	5.673201561	4.668779373	6.217437744
λ = 1.064 μm	7.060328960	8.410255432	7.751673222	9.219581604
	9.651021957	—	9.894903183	—
d = 0.500 μm	1.368159413	2.728582621	1.551661611	3.099793911
ε = 12.25	4.070446014	5.372024536	4.636415958	6.116473675
λ = 1.550 μm	6.552733421	—	6.791202545	—

Note: d = one-half thickness
ε = dielectric coefficient
λ = free space wavelength

4. Calculation of Propagation Parameters

Once we have found values for u, we can calculate k_1, Γ, and β. given k [3] and Δk (the bandwidth of the source), we can also calculate $\Delta\beta$ [4] (the bandwidth in the material). We shall first find $\Delta\beta$ for the TE even mode. Differentiating equations (9), (17), (18), and (19) gives us

$$k_1\, dk_1 + \beta d\beta = \varepsilon k dk \tag{35}$$

$$du = dk_1 d \tag{36}$$

$$dA = dkd\sqrt{\varepsilon - 1} \tag{37}$$

$$\sec^2 u = \frac{A(dA/du) - u}{u\sqrt{A^2 - u^2}} - \frac{\sqrt{A^2 - u^2}}{u^2} \quad . \tag{38}$$

Substituting equation (37) for dA in equation (38) and rearranging yield

$$du = \frac{Adk}{k_1 \sec^2 u(u \tan u + 1)} \quad . \tag{39}$$

There are no unknowns on the right side of the equations. We can now introduce equations (35) and (36) and solve for $d\beta$. We get

$$d\beta = \frac{kdk}{\beta}\left(\varepsilon - \frac{\varepsilon - 1}{\sec^2 u(u \tan u + 1)}\right) \quad . \tag{40a}$$

For the other solution types, we get

$$d\beta = \frac{kdk}{\beta}\left(\varepsilon - \frac{\varepsilon - 1}{\csc^2 u(u \cot u + 1)}\right) \quad \text{TE odd} \tag{40b}$$

$$d\beta = \frac{kdk}{\beta}\left(\varepsilon - \frac{\varepsilon^2(\varepsilon - 1)}{\sec^2 u(u \tan u + 1)}\right) \quad \text{TM even} \tag{40c}$$

$$d\beta = \frac{kdk}{\beta}\left(\varepsilon - \frac{\varepsilon^2(\varepsilon - 1)}{\csc^2 u(u \cot u + 1)}\right) \quad \text{TM odd .} \tag{40d}$$

If we know the propagation parameters k, k_1, Γ, and β, we can find expressions for E_2, E_4, H_2, and H_4 in terms of E_1, E_3, H_1, and H_3. Solving equation (14) for E_2 yields

$$E_2 = E_1 \cos k_1 d e^{\Gamma d} \quad . \tag{41}$$

Similar manipulations give the other desired relationships:

$$E_4 = E_3 \sin k_1 e^{\Gamma d} \tag{42}$$

$$H_2 = H_1 \cos k_1 d e^{\Gamma d} \tag{43}$$

$$H_4 = H_3 \sin k_1 d e^{\Gamma d} \quad . \tag{44}$$

These equations are useful for calculating the electric or magnetic field at any point, which will be done later, and for comparing the energy propagated inside the slab to that outside the slab.

The time-averaged energy flux at any point is defined [5] by

$$<S> = Re\left(\frac{c}{8\pi} \vec{E} x \vec{H}^* \right) \quad . \tag{45}$$

In the TE case, E_z is defined by equation (7) inside the slab, and equation (8) outside the slab. E_x and E_y are both zero. Taking H^* from equations (3) and (4) gives us

$$H_x^* = \frac{\beta}{k} E_1 \cos k_1 x \, e^{-i\beta y} \qquad |x| < d \tag{46}$$

$$H_x^* = \frac{\beta}{k} E_2 \, e^{-\Gamma x} \, e^{-i\beta y} \qquad |x| > d \tag{47}$$

$$H_y^* = \frac{i k_1}{k} E_1 \sin k_1 x e^{-i\beta y} \qquad |x| < d \tag{48}$$

$$H_y^* = \frac{i\Gamma}{k} E_2 \, e^{-\Gamma x} e^{-i\beta y} \qquad |x| > d \quad . \tag{49}$$

Inside the waveguide, equation (45) becomes

$$<S> = \frac{c}{8\pi} E_1^2 \frac{\beta}{k} \cos^2 k_1 x \qquad |x| < d \quad , \tag{50}$$

and outside we have

$$<S> = \frac{c}{8\pi} E_2^2 \frac{\beta}{k} e^{-2\Gamma x} \qquad |x| > d \ . \tag{51}$$

We define the efficiency of the waveguide as

$$eff = \frac{2\int_0^d <S>\, dx}{2\int_0^\infty <S>\, dx} \ . \tag{52}$$

The denominator can be expressed as

$$2\int_0^\infty <S> dx = 2\int_0^d <S> dx + 2\int_d^\infty <S> dx \ . \tag{53}$$

We first calculate the integral from zero to d (energy inside the slab):

$$\begin{aligned} 2\int_0^d <S> dx &= \frac{c}{4\pi} \frac{\beta}{k} E_1^2 \int_0^d \cos^2 k_1 x dx \\ &= \frac{c\beta E_1^2 d}{8\pi k} \left(1 + \frac{\sin 2k_1 d}{2k_1 d}\right) \quad |x| < d \ ; \end{aligned} \tag{54}$$

and outside the slab we have

$$2\int_d^\infty <S> dx = \frac{c}{4\pi} \frac{\beta}{k} E_2^2 \int_d^\infty e^{-2\Gamma x} dx = \frac{c\beta E_2^2}{8\Gamma\pi k} e^{-2\Gamma d} \quad |x| > d \ . \tag{55}$$

Now using equation (41), equation (55) becomes

$$2\int_d^\infty <S> dx = \frac{c\beta E_1^2 \cos^2 k_1 d}{8\Gamma\pi k} \ , \tag{56}$$

and the efficiency of the waveguide becomes

$$eff = \frac{\Gamma d + \sin u}{\Gamma d + 1} \tag{57}$$

for the TE even modes. Similar calculations yield

$$eff = \frac{\Gamma d - \sin^2 u}{\Gamma d} \tag{58}$$

for the TE odd modes. These calculations can also be done for the TM modes, giving us

$$eff = \frac{\varepsilon \Gamma d + \sin^2 u}{\varepsilon \Gamma d + 1} \tag{59}$$

for TM even modes, and

$$eff = \frac{\varepsilon \Gamma d - \sin^2 u}{\varepsilon \Gamma d} \tag{60}$$

for the TM odd modes.

We now have a method for determining the propagation parameters of a waveguide, given its physical properties and the wavelength used. We can also determine the frequency bandwidth inside the material for each mode of operation, given the bandwidth of the incident light. The efficiency of the waveguide is also calculable for each mode. These parameters are presented in table 3(a through j) for various thicknesses, wavelengths, and indices of refraction.

Also presented are plots of E_z (or H_z for the TM modes) versus x. These plots were calculated using, for the TE even case, equations (7), (8), and (41), and assuming that $E_1 = 1$. For the other cases, we used corresponding equations and assumed either $E_3 = 1$, $H_1 = 1$, or $H_3 = 1$. These plots are figures 3 through 8.

An interesting feature of these plots is the bend in each of the TM curves. This means that there must be a discontinuity in the derivative of H_z. This is correct and is caused by the discontinous polarization current density at the surface of the slab. In calculations for the TM modes, E_y must be continuous. In this case, E_y is defined by

$$E_y = \frac{1}{ik\varepsilon} \frac{\partial}{\partial x} H_z \ . \tag{27}$$

At $x = d$, ε has a jump discontinuity. For E_y to be continuous, $\partial/\partial x H_z$ must also have a jump discontinuity.

Interesting conclusions can also be drawn from table 3. We see that $\Delta\beta$ is larger than Δk and, in many cases, much larger. We see that it is inversely proportional to β. We also see that β, as expected, can have a value from k to $(\varepsilon)^{1/2}k$. We can also compare the efficiencies of various modes. Subsequent solutions for one mode type are always less efficient. The first TE even solution is always more efficient than the second TE even solution, and so on. This is true for all four mode types. Also, the TM modes are always more efficient than their corresponding TE modes.

The trends evidenced in the sample data cannot necessarily be generalized for all situations.

Table 3. Propagation Parameters for Slab Waveguides

Mode	k_1	Γ	β	$\Delta\beta$	efficiency
A. Thickness = 1.00000 μm, ε = 4.800, wavelength = 1.06400 μm, bandwidth = 0.00100 μm, *k* = 59052, and Δ *k* = 55.501					
TEE	26729	111968	126586	123.47	0.99183
TEO	53220	102074	117925	138.91	0.95812
TME	30281	111060	125784	125.07	0.99988
TMO	60284	98067	114474	137.43	0.99931
TEE	79097	83636	102382	142.57	0.90889
TEO	103363	50669	77811	286.33	0.68176
TME	89241	72714	93672	167.92	0.99667
TMO	111714	27773	65257	241.68	0.93811
B. Thickness = 0.10000 μm, ε = 12.250, wavelength = 0.82000 μm, bandwidth = 0.00001 μm, *k* = 76624, and Δ *k* = 0.934					
TEE	169806	192919	207579	3.36	0.77780
TME	249663	60993	97936	8.95	0.97879
C. Thickness = 0.10000 μm, ε = 12.250, wavelength = 1.06400 μm, bandwidth = 0.00010 μm, *k* = 59052, and Δ *k* = 5.550					
TEE	146960	132792	145331	19.23	0.66916
TME	196598	24084	63775	62.91	0.87576

Table 3. Propagation Parameters for Slab Waveguide (cont'd)

Mode	k_1	Γ	β	$\Delta\beta$	efficiency
D. Thickness = 0.10000 μm, ε = 12.250, wavelength = 1.55000 μm, bandwidth = 0.00400 μm, *k* = 40537, and Δ *k* = 104.611					
TEE	114343	73567	83996	324.79	0.48295
TME	135671	8917	41506	1248.57	0.60792
E. Thickness = 0.35000 μm, ε = 12.250, wavelength = 0.82000 μm, bandwidth = 0.00001 μm, *k* = 76624, and Δ *k* = 0.934					
TEE	73245	246347	257989	3.35	0.98471
TEO	145209	212052	225472	4.31	0.91398
TME	88059	241449	253315	3.46	0.99998
TMO	175182	188050	203062	4.32	0.99986
TEE	213326	143331	162527	4.42	0.80362
TME	250183	58822	96599	9.08	0.99210
F. Thickness = 0.35000 μm, ε = 12.250, wavelength = 1.06400 μm, bandwidth = 0.00010 μm, *k* = 59052, and Δ *k* = 5.550					
TEE	69326	185539	194710	20.07	0.97115
TEO	136194	143813	155465	33.22	0.81213
TME	87465	177710	187265	21.44	0.99996
TMO	171500	99090	115352	34.81	0.99908
TEE	192768	45512	74556	27.78	0.47274
TME	197994	5415	59300	67.53	0.58392
G. Thickness = 0.35000 μm, ε = 12.250, wavelength = 1.55000 μm, bandwidth = 0.00400 μm, *k* = 40537, and Δ *k* = 104.611					
TEE	62477	120759	127382	382.41	0.93218
TEO	118791	66143	77577	3650.06	0.34053
TME	85959	105343	112874	460.23	0.99981
TMO	135486	11391	42107	1236.25	0.80131

Table 3. **Propagation Parameters for Slab Waveguide (cont'd)**

Mode	k_1	Γ	β	$\Delta\beta$	efficiency
H. Thickness = 1.00000 μm, ε = 12.250, wavelength = 0.82000 μm, bandwidth = 0.00001 μm, *k* = 76624, and Δ *k* = 0.934					
TEE	29143	255348	266597	3.29	0.99907
TEO	58258	250315	261780	3.36	0.99589
TME	31216	255103	266362	3.29	1.00000
TMO	62423	249309	260819	3.36	1.00000
TEE	87315	241719	253573	3.43	0.99118
TEO	116274	229199	241668	3.69	0.98214
TME	93609	239351	251317	3.49	0.99999
TMO	124758	224694	237400	3.69	0.99999
TEE	145083	212139	225553	3.79	0.97254
TEO	173657	189460	204368	4.50	0.95180
TME	155836	204369	218262	4.02	0.99997
TMO	186773	176544	192455	4.56	0.99993
TEE	201845	159092	176583	4.65	0.93112
TEO	229286	116102	139108	7.26	0.86289
TME	217339	137170	157120	5.58	0.99981
TMO	246044	74256	106701	8.22	0.99850
TEE	254256	37494	85306	7.07	0.65954
TME	256934	6041	76862	11.40	0.80353
I. Thickness = 1.00000 μm, ε = 12.250, wavelength = 1.06400 μm, bandwidth = 0.00010 μm, *k* = 59052, and Δ *k* = 5.550					
TEE	28526	196003	204706	19.58	0.99808
TEO	56994	189691	198670	20.39	0.99127
TME	31156	195602	204322	19.65	1.00000
TMO	62291	188018	197074	20.37	0.99999
TEE	85339	178741	188243	20.96	0.98132
TEO	113464	162348	172754	24.22	0.95957
TME	93376	174677	184389	21.77	0.99998
TMO	124349	154170	165093	24.32	0.99995
TEE	141207	138894	150927	25.04	0.93603
TEO	168205	104585	120105	38.66	0.86209
TME	155033	123270	136685	29.37	0.99986
TMO	184392	72323	93370	43.00	0.99906
TEE	193020	44431	73900	39.62	0.70521
TME	197898	8205	59620	67.29	0.86810

Table 3. Propagation Parameters for Slab Waveguide (cont'd)

Mode	k_1	Γ	β	$\Delta\beta$	efficiency
J. Thickness = 1.00000 μm, ε = 12.250, wavelength = 1.55000 μm, bandwidth = 0.00400 μm, k = 40537, and Δ k = 104.611					
TEE	27363	133182	139215	371.33	0.99471
TEO	54572	124532	130963	407.88	0.97413
TME	31033	132375	138443	375.23	1.00000
TMO	61996	121007	127617	407.06	0.99998
TEE	81409	108898	116198	424.22	0.94437
TEO	107441	83323	92661	662.16	0.85012
TME	92728	99437	107382	483.76	0.99991
TMO	122329	59344	71868	722.82	0.99924
TEE	131055	36207	54352	665.58	0.66940
TME	135824	6171	41004	1265.66	0.84028

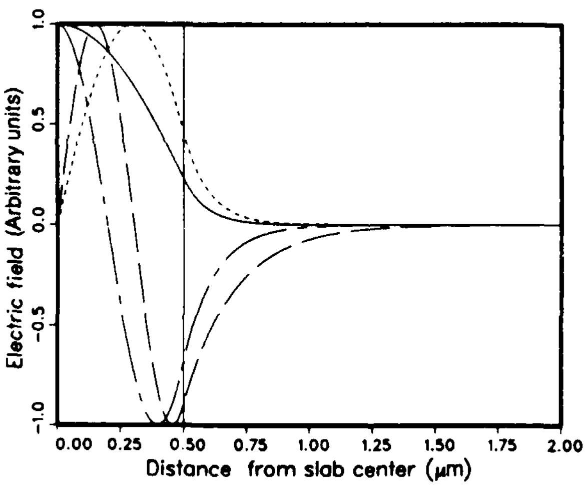
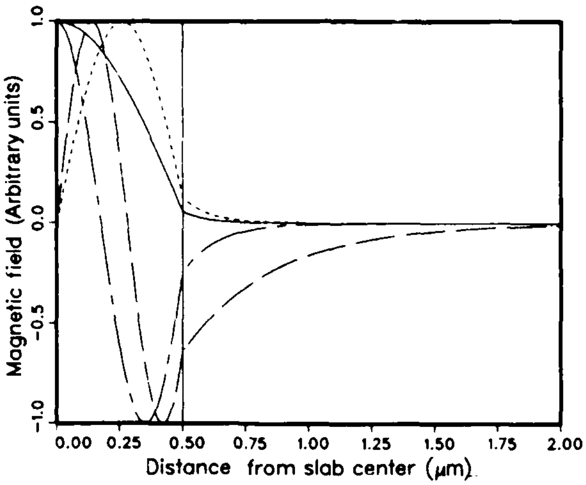
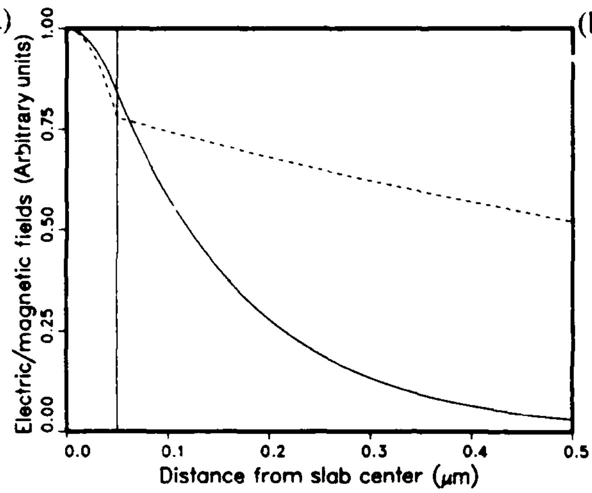


Figure 3. Electric field (arbitrary units) versus distance from slab center for two even and two odd TE modes, for a slab of 1 μm thickness, a dielectric coefficient of 4.80, and a free space wavelength of 1.064 μm. Vertical line in this, and all following figures, represents slab edge.

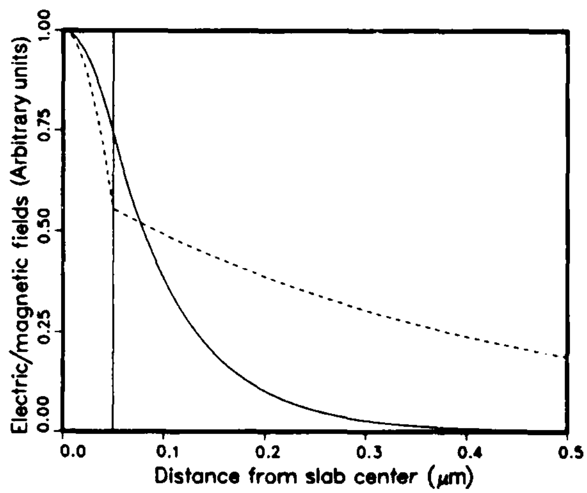
Figure 4. Magnetic field (arbitrary units) versus distance from slab center for two even and two odd TM modes. All parameters are same as for figure 3.



(a)



(b)



(c)

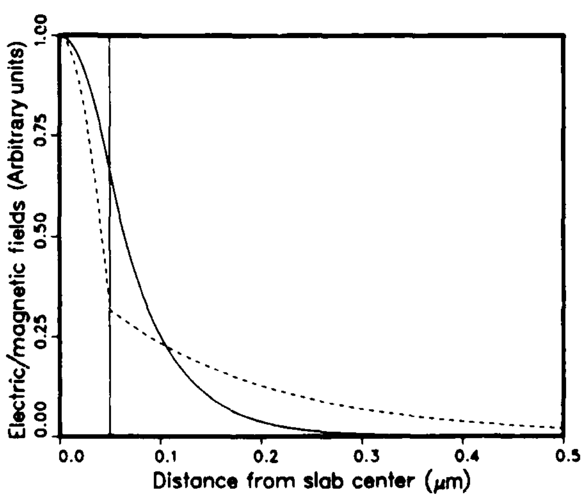
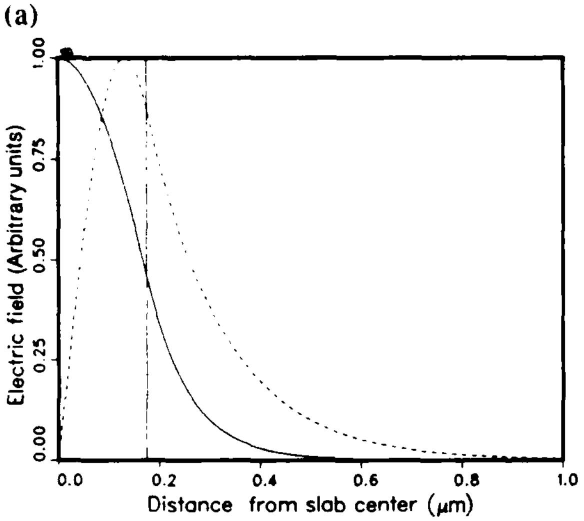


Figure 5. Electric field for TE even mode and magnetic field for TM even mode (only modes possible) for a slab of 0.1 μm thickness and a dielectric coefficient of 12.25. Free space wavelength is (a) 1.550, (b) 1.064, and (c) 0.820 μm.



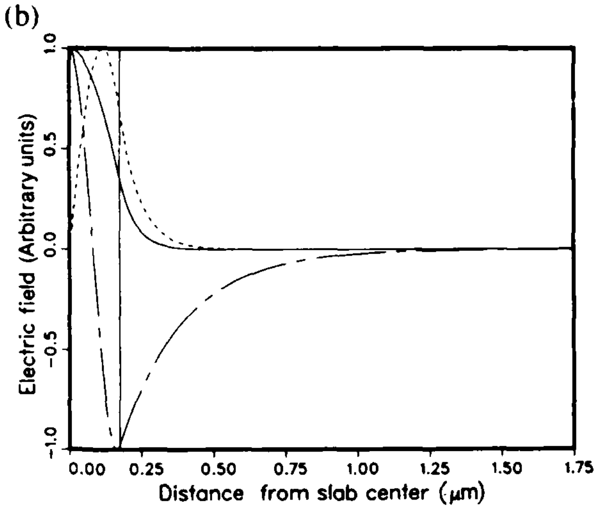
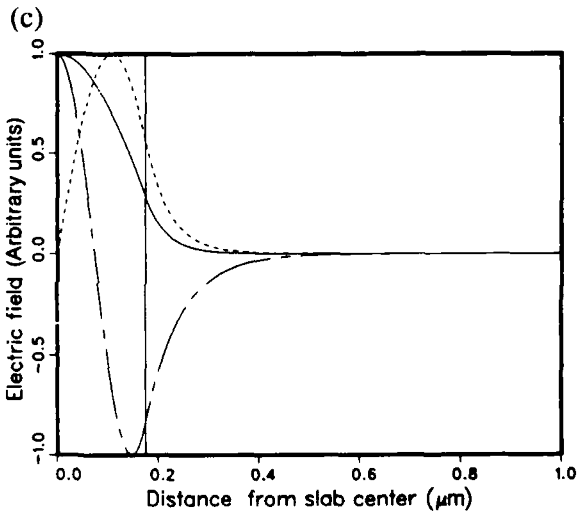


Figure 6. Electric field versus distance from slab center. Slab thickness of 0.35 μm, a dielectric coefficient of 12.25, and free space wavelength of (a) 1.550 μm, for one even mode and one odd mode; (b) 1.064 μm, for two even modes and one odd mode; and (c) 0.820 μm, for two even modes and one odd mode.



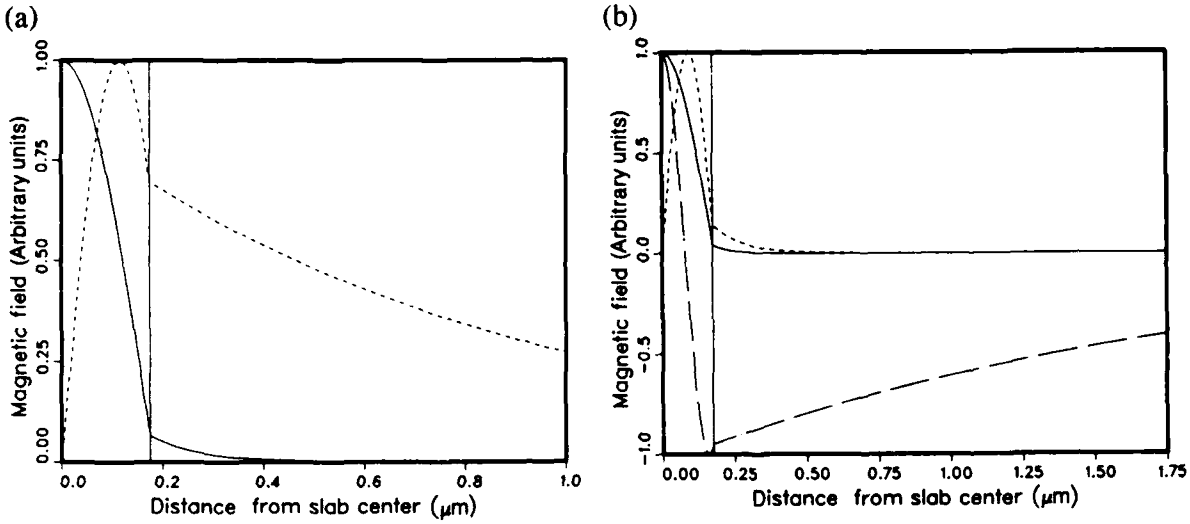


Figure 7. Magnetic field versus distance from slab center. Slab has thickness of 0.35 μm, a dielectric coefficient of 12.25, and free space wavelength of (a) 1.550 μm, for one even mode and one odd mode; (b) 1.064 μm, for two even modes and one odd mode; and (c) 0.820 μm, for two even modes and one odd mode.

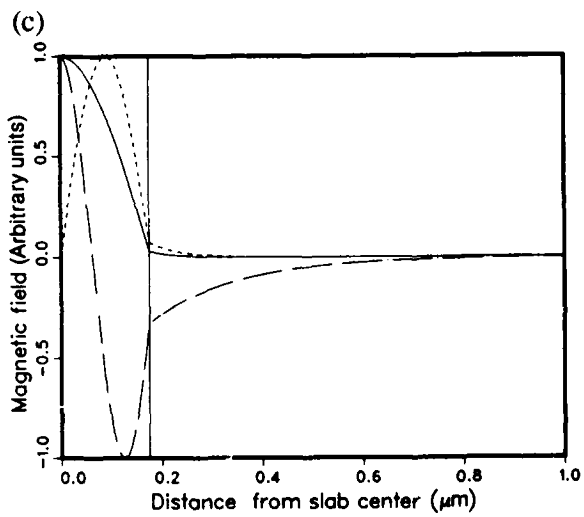
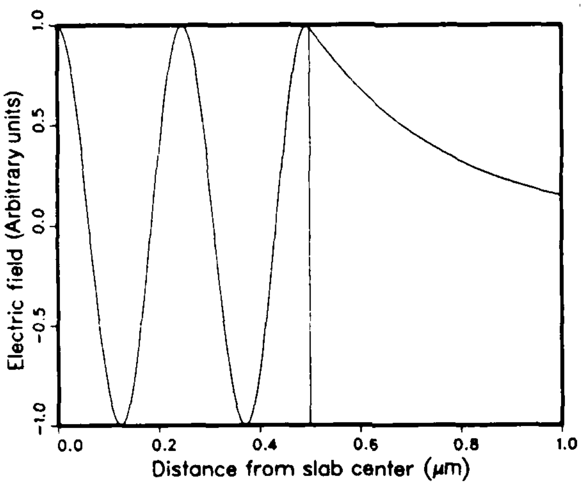


Figure 8. Highest (fifth) TE even mode for a slab with a thickness of 1 μm and a dielectric coefficient of 12.25, for a free space wavelength of 0.820 μm.



Acknowledgements

Thanks to Clyde Morrison for guidance and direction.

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Appendix A.--FORTRAN Programs Used in Calculating Dielectric Waveguide Characteristics

```
C$FORT SLAB1
C$LINK SLAB1
C$PURGE SLAB1.EXE
C$DEL SLAB1.OBJ;*
C
C               THIS PROGRAM FINDS THE SOLUTIONS FOR AN
C               INFINITE PLANAR SLAB WAVEGUIDE WITH DIELECTRIC
C               CONSTANT EPS, THICKNESS THK, AND INCIDENT
C               WAVE LENGTH WL.
C
        PROGRAM NEWRAP
        CHARACTER*1 ANS
        DIMENSION TEE(500),TEO(500),TME(500),TMO(50C)
        TYPE *,'  THIS PROGRAM SOLVES FOR THE'
        TYPE*,'  ROOTS TO OUR SLAB WAVEGUIDE PROBLEM.'
        TYPE*,' '
C       TYPE*,' ENTER DIELECTRIC CONSTANT,THICKNESS,WAVE LENGTH'
C
C       THIS FILE CONTAINS THE RUN PARAMETERS.  IT IS MADE BY THE USER.
C       YOU MUST STORE THEM IN THE FILE LIKE THIS:
C       EPSILON THICKNESS(cm) WAVELENGTH(cm) BANDWIDTH(cm)
C
        OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
        READ(9,*) , EPS , THK,WL,BW
        THK=THK/2
C
C      SET RUN CONSTANTS
C
        EFS=1.0
        PI=3.141592554
        A=2*PI*THK*(EPS-1)**.5/WL
        IO=IFIX(A/PI+.5)
C
C       IF A IS LESS THEN PI/2 THEN WE HAVE ONLY TWO ROOTS, ONE TE EVEN
C       AND ONE TM EVEN.  THERE WILL BE NO ODD ROOTS, SO WE GO TO THE
C       SPECIAL EVEN ONLY ROUTINE.
C
        IF(A.LT.PI/2)GOTO 123
100     FORMAT(16x,4F16.9)
C
C      START LOOP, AND SET INITIAL GUESSES FOR THE NEWTON-RAPHSON
C       ROUTINE
C
        I=1
        SE=PI/2.0-THK
        SE1=SE
        SO=PI-THK
C
C       IF OUR GUESS IS GREATER THAN A, THE PROGRAM WILL CRASH.  SO WE
C       INSURE THAT OUR GUESSES ARE LESS THAN A.
        IF (SO .GT. A)SO=A-THK
        SO1=SO
C
C      USE SUBROUTINES TO FIND ROOTS
C      EPS IS EPSILON, EFS IS EPSILON OF FREE SPACE
C
C       THERE SHOULD BE ONE SOLUTION OF EACH TYPE - TWO ODD, TWO EVEN,
C       OF TM AND TE MODES.
C
10      IF (SO .GE. A) SO=A-THK
        IF (SO1 .GE. A) SO1=SO
        CALL ODD(TEO(I),SO,EFS,A)
        CALL ODD(TMO(I),SO1,EPS,A)
        CALL EVEN(TEE(I),SE,EFS,A)
        CALL EVEN(TME(I),SE1,EPS,A)
C
C     SET UPPER AND LOWER LIMITS FOR EACH INDIVIDUAL MODE
C
        AI=PI*(2*I+1)/2.0
        AJ=PI*(2*I-1)/2.0
        AL=PI*(2*I-3)/2.0
C
```

```
C     CHECK THAT ROOT IS WITHIN THE SET LIMITS
C     THAT IS, CHECK THAT NO ROOTS ARE SKIPPED, OR REPEATED.
C
        IF (TEO(I) .GT. AI .OR. TMO(I) .GT. AI) IFLG=IFLG+1
        IF (TEE(I) .GT. AJ .OR. TME(I) .GT. AJ) JFLG=JFLG+1
        IF (TEO(I) .LT. AJ .OR. TMO(I) .LT. AJ) KFLG=KFLG+1
        IF (TEE(I) .LT. AL .OR. TME(I) .LT. AL) LFLG=LFLG+1
C
        TYPE 100,TEE(I),TEO(I),TME(I),TMO(I)
123     I=I+1
C
C     CHECK TO SEE IF WE'RE DONE.
C
        IF (I .LE. 10) GOTO 10
C
C     CHECK TO SEE IF WE HAVE ANOTHER  PAIR OF EVEN ROOTS.
C
        IF (A-(I-1)*PI .LE. 0) GOTO 987
C
C       FIND PAIR OF EVEN ROOTS
C
        SE=A-THK
        SE1=SE+THK/2
C       TYPE*,I0*PI,SE,A
        CALL EVEN(TEE(I),SE,EFS,A)
        CALL EVEN(TME(I),SE1,EPS,A)
        IXFL=1
C
C       IF ANY OF THE FLAGS BELOW ARE NOT ZERO, A MODE WAS SKIPPED
C       OR REPEATED,
C
987     TYPE *,IFLG,JFLG,KFLG,LFLG

200     FORMAT(A1)
C
C     OPEN FILE FOR EZ-GRAPH PLOTTING, AND FILE FOR PRINT OUT
C
        OPEN(UNIT=2,TYPE='NEW',NAME='ALROOT.EZG')
        OPEN (UNIT=1,TYPE='NEW',NAME='ALROOT.DAT')
        WRITE(1,*) ' '
        DO 20 I=1,I0,1
        WRITE(1,100) TEE(I),TEO(I),TME(I),TMO(I)
C
C       CALCULATE THE VALUES FOR THE PLOTTING OF THE INTERSECTION
C       OF THE TRANSCENDENTAL EQUATIONS.
C
        CN=-1.0/TAN(TEO(I))
        CN1=-1.0/TAN(TMO(I))
        TN=TAN(TEE(I))
        TN1=TAN(TME(I))
C
C       WRITE TO EZG FILE
C
        WRITE(2,*) TEE(I),TN
        WRITE(2,*) TEO(I),CN
        WRITE(2,*) TME(I),TN1
        WRITE(2,*) TMO(I),CN1
20      CONTINUE
        IF (IXFL .NE. 1) GOTO 789
C       WRITE(2,*) 0.0,99999.
        WRITE(1,234) TEE(I),TME(I)
234     FORMAT(16X,F16.9,'                ',F16.9)
        TN=TAN(TEE(I))
        WRITE(2,*) TEE(I),TN
        TN=TAN(TME(I))
        WRITE(2,*) 0.0,99999.
        WRITE(2,*) TME(I),TN
789     STOP
        END
        SUBROUTINE ODD(F,X,E,A)
C
C        THIS SUBROUTINE FINDS THE ROOTS OF ODD SOLUTION MODES
```

```
C       DEPENDING ON E, IT WILL SOLVE FOR TMO, OR TME
C
        DOUBLE PRECISION Q,F2,F3,D1
320     Q=1/(A*A-X*X)**.5
C       TYPE*,X,D1
        F2=TAN(X)+X*Q/E
        F3=COS(X)**(-2)+Q/E+X*X*Q**3/E
        D1=-F2/F3
        IF (ABS(D1) .LE. .00001)GOTO 310
        X=X+D1
        GOTO 320
310     F=X
        X=X+3.141592554
        RETURN
        END
        SUBROUTINE EVEN(F,X,E,A)
C
C      THIS SUROUTINE FINDS THE ROOTS FOR EVEN SOLUTION MODES
C
        DOUBLE PRECISION Q,F2,F3,D1
420     Q=(A*A-X*X)**.5
C       TYPE*,X,D1
        F2=TAN(X)-Q*E/X
        F3=COS(X)**(-2)+Q*E/(X*X)+E/Q
        D1=-F2/F3
        IF (ABS(D1) .LE. .00001) GOTO 410
        X=D1+X
        GOTO 420
410     F=X
        X=X+3.141592554
        RETURN
        END
```

```
C$FORT SLAB2
C$LINK SLAB2
C$DEL SLAB2.OBJ;*
C$PURGE SLAB2.EXE
C
C       THIS PROGRAM CALCULATES THE PROPAGATION PARAMETERS FOR THE
C       INDIVIDUAL SOLUTIONS TO THE SLAB WAVEGUIDE PROBLEM.
        PROGRAM BETAWD
        CHARACTER*5 MD(4)
        REAL K,K1,EFF(4)
        DATA PI/3.1415926536/
C
C      THESE CHARACTER STRINGS REPRESENT THE FOUR DIFFERENT MODE TYPES.
C
        MD(1)=' TEE '
        MD(2)=' TEO '
        MD(3)=' TME '
        MD(4)=' TMO '
C
C       OPEN THE RUN PARAMETER FILE(CREATED BY THE USER).
        OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
C
        READ(9,*) EPS,THK,WL,DW
        DK=2*PI*DW/(WL**2-DW**2)
        D=THK/2
        K=2*PI/WL
C
C       OPEN SOLUTION DATA FILE, CREATED BY SLAB1 PROGRAM)
        OPEN (UNIT=2,TYPE='OLD',NAME='ALROOT.EZG')
C
C       OPEN OUTPUT FILE FOR PROPAGATION PARAMETERS, AND
C       WRITE FILE HEADER
        OPEN (UNIT=1,TYPE='NEW',NAME='PARAMS.DAT')
        WRITE(1,*)' Table .  Propogation parameters for slab waveguide
     + with'
        WRITE(1,*) ' '
        WRITE(1,10) THK*10000,EPS
10      FORMAT('         thickness=',F7.5,' um','     epsilon=',F7.3)
        WRITE(1,12) WL*10000,DW*10000
12      FORMAT('        wavelength=',F7.5,' um','  bandwidth=',
     +F7.5,' um')
        WRITE(1,14) K,dk
14      FORMAT('                 k=',F7.0,'       delta k=',F7.3)
        write(1,*)'__________________________________________________
     +___________________________'
        WRITE(1,*)' '
        WRITE(1,*) 'mode     k          gamma      beta
     + delta beta  efficiency  '
        write(1,*) '           1'
        write(1,*)'__________________________________________________
     +___________________________'
999     FORMAT(A5,3F10.0,F11.2,F13.5)
C
C       MODE TYPE IS KEPT TRACK OF WITH MTP(0 BEFORE PROGRAM STARTS)
        MTP=0
100     READ(2,*,END=777) U,DUMMY
        MTP=MTP+1
        IF (MTP .GT. 4) MTP=MTP-4
        IF(U.EQ. 0) GOTO 100
C
C       CALCULATE K1,GAMMA
        K1=U/D
        GAM=(K*K*(EPS-1)-K1*K1)**.5
C
C       CALCULATE EFFICIENCY AND BETA "B"
        B=(GAM**2+K*K)**.5
        SU=SIN(U)
        CU=COS(U)
        SC=SU*CU
        GD=GAM*D
        GED=GD*EPS
        EFF(1)=(GD+SU**2)/(GD+1)
```

```
      EFF(2)=(GD-SU**2)/GD
      EFF(3)=(GED+SU**2)/(GED+1)
      EFF(4)=(GED-SU**2)/GED
C
C     CALCULATE THE BANDWIDTH OF BETA
      CSU2=1/SU**2
      SCU2=1/CU**2
      tu=su/cu
      ct=1/tu
      Q=1
      IF (MTP .GT. 2) Q=1/EPS**2
      if (mtp .eq. 2*int(mtp/2)) goto 987
      f1=scu2
      f2=tu
      goto 986
987   f1=csu2
      f2=ct
986   db=k*dk*(eps-q*(eps-1)/(f1*(u*f2+1)))/b
C
C     WRITE TO OUTPUT FILE
      WRITE(1,999) MD(MTP),K1,GAM,B,DB,EFF(MTP)
C
      GOTO 100
777   write(1,*)'_______________________________________
     +_____________________'
      CLOSE(UNIT=1)
      STOP
      END
```

```
C$FORT SLAB3
C$LINK SLAB3
C$DEL SLAB3.OBJ;*
C$PURGE SLAB3.EXE
C
C       THIS PROGRAM CALCULATES THE VALUES OF THE CURVES REPRESENTED
C       BY THE FOUR TRANSCENDENTAL EQUATIONS FOR OUR SLAB WAVEGUIDE
C       PROBLEM. THE FOUR CURVES NEEDED ARE:
C
C     F1(U)=TAN(U)  F2(U)=-COT(U)  F3(U)=(A*A-U*U)**.5  F4(U)=EPS*F(3)
C
        PROGRAM RTGRA
        DIMENSION F(4)
        DATA U,WL,PI/0.0,.000106,3.1415926536/
C
C    OPEN RUN PARAMETER FILE (CREATED BY THE USER) AND READ PARAMETERS
        OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
        READ(9,*) EPS,THK,WL,BW
C
C       CALCULATE D(HALF THICKNESS) AND A(DESCRIBED IN PAPER)
        D=THK/2
        A=2*PI*D*(EPS-1)**.5/WL
        N=0
        DU-PI/2000.
C
C       OPEN FILE FOR OUTPUT OF CURVES.  THE OUTPUT IS OF THE FORM:
C       U  F(1) F(2) F(3) F(4)
C
9       OPEN(UNIT=1,TYPE='NEW',NAME='SMROOT.DAT')
C
C       BEGIN CURVE CALCULATING LOOP
        DO 10 I=0,999
C
C       WE CALCULATE IN BLOCKS OF PI/2 TO AVOID A VERTICLE LINE WHEN
C       WE PLOT THE TRIGONOMETRIC CURVES WITH EZGRAPH.
        U=I*DU+N*PI/2.
        IF(U .GT. A) GOTO 11
        IF (INT(2*U/PI) .EQ. 2*U/PI) GOTO 10
        F(1)=TAN(U)
        F(2)=-1/F(1)
        F(3)=(A*A-U*U)**.5/U
        F(4)=EPS*F(3)
        DO 20 J=1,4
        IF(ABS(F(J)) .LT. 100) GOTO 20
C
C    FOR PLOTTING PURPOSES, WE DONT WANT ANY OF THE FUNCTIONS VERY HIGH
C    IT DOESN'T MATTER MUCH EITHER WAY
        F(J)=99.9999*ABS(F(J))/F(J)
20      CONTINUE
        WRITE(1,100) U,F(1),F(2),F(3),F(4)
10      CONTINUE
100     FORMAT(5F15.10)
        N=N+1
11      CLOSE(UNIT=1)
        IF(U .LT. A ) GOTO 9
        STOP
        END
```

```
C$FORT SLAB4
C$LINK SLAB4
C$DEL SLAB4.OBJ;*
C$PURGE SLAB4.EXE
C
C               THIS PROGRAM CALCULATES THE VALUES OF THE ELECTRIC
C               OR MAGNETIC FIELDS FOR ANY X, INSIDE OR OUTSIDE THE
C               SLAB.  IT ASSUMES A MAXIMUM POSSIBLE FIELD OF ONE, OR
C               E1=1, AND Y=2*PI*n.
        PROGRAM MODEPL
        REAL K,K1
        DATA PI/3.1415926536/
        COMMON U,D,K,K1,B,GAM,EGD,EPS,CU,SU,EO
C
C     OPEN FILE CREATED BY USER WITH RUN PARAMETERS
        OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
        READ(9,*) EPS,THK,WL,BW
C
C     CALCULATE PARAMETERS USED FOR FINDING FIELDS
        D=THK/2
        K=2*PI/WL
C
C     OPEN FILE WITH SOLUTIONS TO TRANSCENDENTALS,CONTAINING VALUES FOR U
C     THIS FILE WAS CREATED BY SLAB1
        OPEN (UNIT=2,TYPE='OLD',NAME='ALROOT.EZG')
C
C     KEEP TRACK OF MODE TYPE WITH MTP
        MTP=-1
100     READ(2,*,END=777) U,DUMMY
        MTP=MTP+1
        IF(U.EQ. 0) GOTO 100
        CU=COS(U)
        SU=SIN(U)
C
C     CALCULATE PROPAGATION PARAMETERS
        K1=U/D
        GAM=(K*K*(EPS-1)-K1*K1)**.5
        B=(GAM**2+K*K)**.5
        EGD=EXP(GAM*D)
C
C     OPEN FILE FOR OUTPUT
        OPEN(UNIT=1,TYPE='NEW',NAME='SMMODE.DAT')
        DX=10.*D/1000.
C
C    LOOP TO CALCULATE FIELDS
        DO 200 I=0,1000
        X=I*DX
        EO=MTP-2*IFIX(FLOAT(MTP)/2.0)
        CALL MODCLC(X,V)
        WRITE(1,*)X*10000,V
200     CONTINUE
        CLOSE(UNIT=1)
        GOTO 100
777     STOP
        END
        SUBROUTINE MODCLC(X,V)
C
C    THIS SUBROUTINE CALCULATES THE FIELD DEPENDING ON WHICH TYPE OF
C    IS BEING PLOTTED
        REAL K,K1
        COMMON U,D,K,K1,B,GAM,EGD,EPS,CU,SU,EO
C
C    CHECK FOR INSIDE OR OUTSIDE OF SLAB
        IF (X .LT. D) GOTO 9
```

```
C
C    CHECK FOR EVEN OR ODD MODE
          IF(EO) 2,2,1
2         AM=CU
          GOTO 3
1         AM=SU
3         V=AM*EGD*EXP(-X*GAM)
          GOTO 4

9         IF (EO) 6,6,5
6         V=COS(K1*X)
          GOTO 4
5         V=SIN(K1*X)
4         RETURN
          END
```

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